Probing Primordial Non-Gaussianities with Large-Scale Structure Data

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Cosmological Non-Gaussianity: Observations Confront Theory

Primordial non-Gaussianity as a new route to falsify Inflation

→ Historical remarks

- Groth and Peebles 1977 (3-pt function)
- Strongly non-Gaussian initial conditions studied in the eighties
- Determination of bispectrum for PSCz galaxies (Fedman et al. 2001) 2dF galaxies (Verde et al. 2002)
- New era with f_{NL} models from inflation (Salopek & Bond 1991; Gangui et al. 1994: $f_{NL} \sim 10^{-2}$; Verde et al. 1999; Komatsu & Spergel 2001; Acquaviva et al. 2002; Maldacena 2002; + many models with higher f_{NL}).
- Primordial NG emerged as a new "smoking gun" of (non-standard) inflation models, which will very soon complement the search for primordial GW

... and to test the physics of the Early Universe

- The NG amplitude and shape measures deviations from standard inflation, perturbation generating processes after inflation, initial state before inflation, ...

- Inflation models which would yield the same predictions for scalar spectral index and tensor-to-scalar ratio might be distuinguishable in terms of NG features.

- We should aim at “reconstructing” the inflationary action, starting from measurements of a few observables (like n_S, r, n_T, f_{NL}, g_{NL}, etc. ...), just like in the nineties we were aiming at a reconstruction of the inflationary potential.

Non-Gaussianity in the initial conditions

a simple-minded NG model … has become reality

✓ Many primordial (inflationary) models of non-Gaussianity can be represented in configuration space by the simple formula (Salopek & Bond 1990; Gangui et al. 1994; Verde et al. 1999; Komatsu & Spergel 2001)

$$\Phi = \phi_L + f_{NL} * (\phi_L^2 - \langle\phi_L^2\rangle) + g_{NL} * (\phi_L^3 - \langle\phi_L^2\rangle \phi_L) + \ldots$$

where Φ is the large-scale gravitational potential, ϕ_L its linear Gaussian contribution and f_{NL} is the dimensionless *non-linearity parameter* (or more generally *non-linearity function*). The percent of non-Gaussianity in CMB data implied by this model is

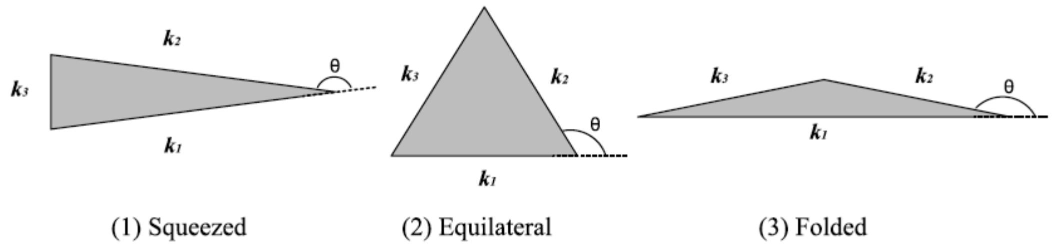
$$\mathrm{NG}\,\% \sim 10^{-5} \; |f_{NL}|$$

$$\sim 10^{-10} \; |g_{NL}|$$

< 10^{-4} from CMB & LSS

< 10^{-4} from CMB & LSS

but .. there are more shapes of non-Gaussianity (from inflation) than ... stars in the sky



- The <u>local</u> shape of NG (→ squeezed triangles in k-space) typically arises in multi-field inflation models (e.g. curvaton, inhomogeneous reheating, etc...)

- Large NG with <u>equilateral</u>, orthogonal, <u>flattened</u> (folded) shapes, etc.. are typical of (non-standard) single-field inflation (Bartolo, Fasiello, Matarrese & Riotto 2010) → *no need for "exotic" initial states to get flattened shape!*

- General (non-separable) CMB bispectra can be expanded in terms of "separable" bispectra (Fergusson, Liguori & Shellard 2010) → general analyis

- <u>Statistical anisotropic</u> NG typically arises if (non-)Abelian vector field are present during inflation (see review by Dimastrogiovanni et al. 2010)

WMAP constraints

	WMAP 7-yrs (Komatsu et al. 2010)
Local	$-10 < f_{NL} < 74$
Equilateral	$-214 < f_{NL} < 266$
Orthogonal	$-410 < f_{NL} < 6$

(95% c.l)

Non-Gaussianity & the LSS

(= primordial NG + NG from gravitational instability)

NG and LSS

- ✓ NG in LSS (to make contact with the CMB definition) can be defined through a potential Φ defined starting from the DM density fluctuation δ through Poisson's equation (use comoving gauge for density fluctuation, Bardeen 1980)

$$\delta = -\left(\frac{3}{2}\Omega_m H^2\right)^{-1} \nabla^2\Phi$$

- ✓ Many primordial (inflationary) models of non-Gaussianity can be represented in configuration space by the simple formula

$$\Phi = \phi_L + f_{NL}(\phi_L^2 - \langle\phi_L^2\rangle) + g_{NL}(\phi_L^3 - \langle\phi_L^2\rangle\phi_L) + \ldots$$

Φ on sub-horizon scales reduces to minus the large-scale gravitational potential, ϕ_L is the linear Gaussian contribution and f_{NL} and g_{NL} are dimensionless non-linearity parameters (or more generally non-linearity functions). CMB and LSS conventions differ by a factor 1.3 for f_{NL}, $(1.3)^2$ for g_{NL}

NG effects in LSS (mass)

- Bartolo, Matarrese & Riotto (2005) computed the effects of NG in the dark matter density fluctuations in a matter-dominated universe. Only for high values of f_{NL} (~10) the standard parameterization is valid. For smaller primordial NG strength non-Newtonian gravitational terms shift f_{NL} by a term ~ - 1.6 (see Verde & Matarrese 2010). On small scales stagnation effects during radiation dominance have to be taken into account up to second order. (Bartolo, Matarrese & Riotto 2007; Senatore et al. 2009).

- Sefusatti & Komatsu (2007) show that LSS becomes competitive with CMB at z > 2.

- ***but .. mass NG is not (all) we measure with galaxy NG***

NG effects on the matter PS: local shape

Calculation based on Renormalization Group (RG) (Matarrese & Pietroni 2007; Pietroni 2008) technique

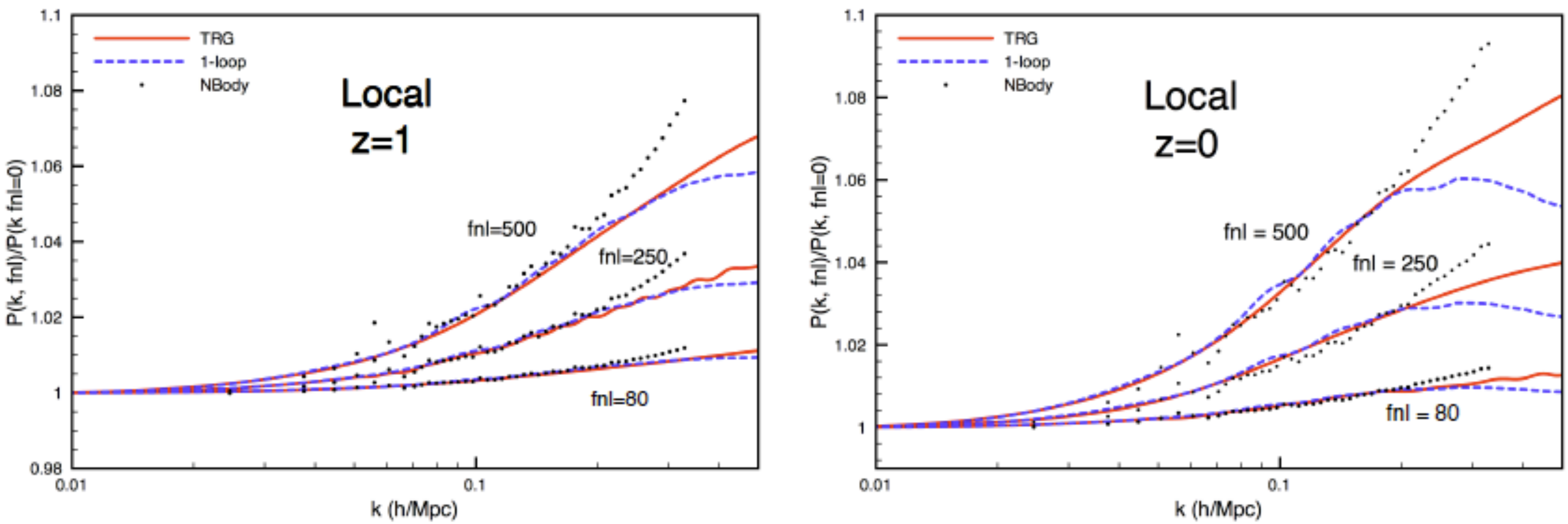


Figure 5. Ratio of the non-Gaussian to Gaussian power spectrum for several values of f_{NL} in the local model. The dots correspond to the data from the N-body simulations of [54]. The red (continuous) line is the TRG result of this paper and the blue (dashed) line is the one-loop result.

Bartolo, Beltrán Almeida, Matarrese, Pietroni & Riotto 2010

NG effects on the matter PS: equilateral and folded shapes

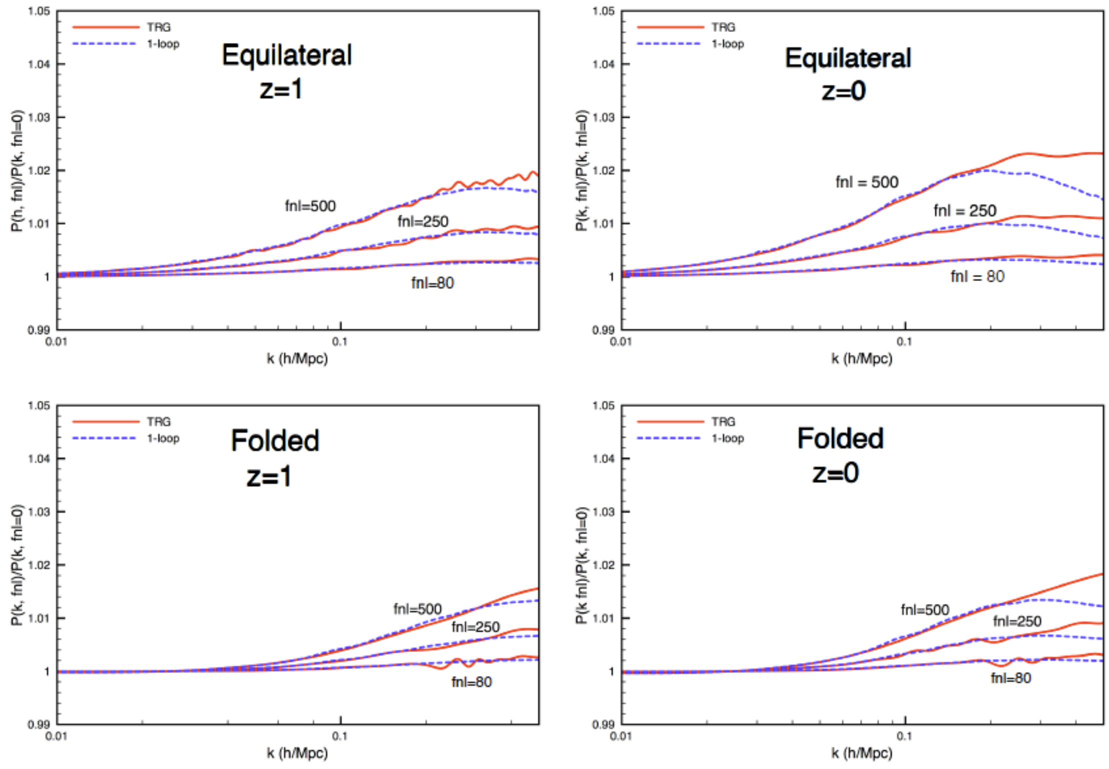
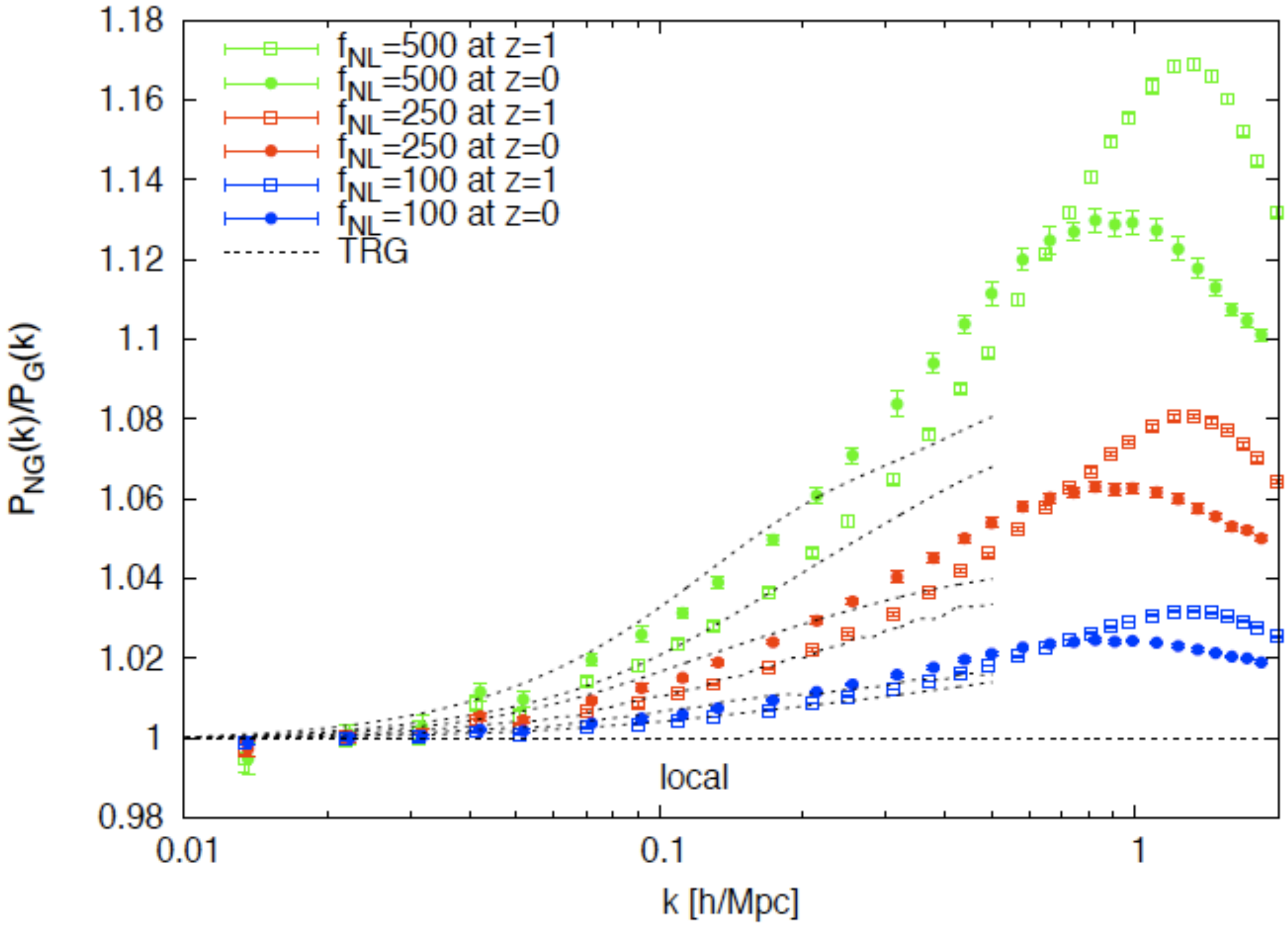


Figure 6. Ratio of the non-Gaussian to Gaussian power spectrum for several values of f_{NL} in the equilateral (top panels) and folded (bottom panels) models. The red (continuous) lines are the TRG result of this paper and the blue (dashed) lines are the one-loop result.

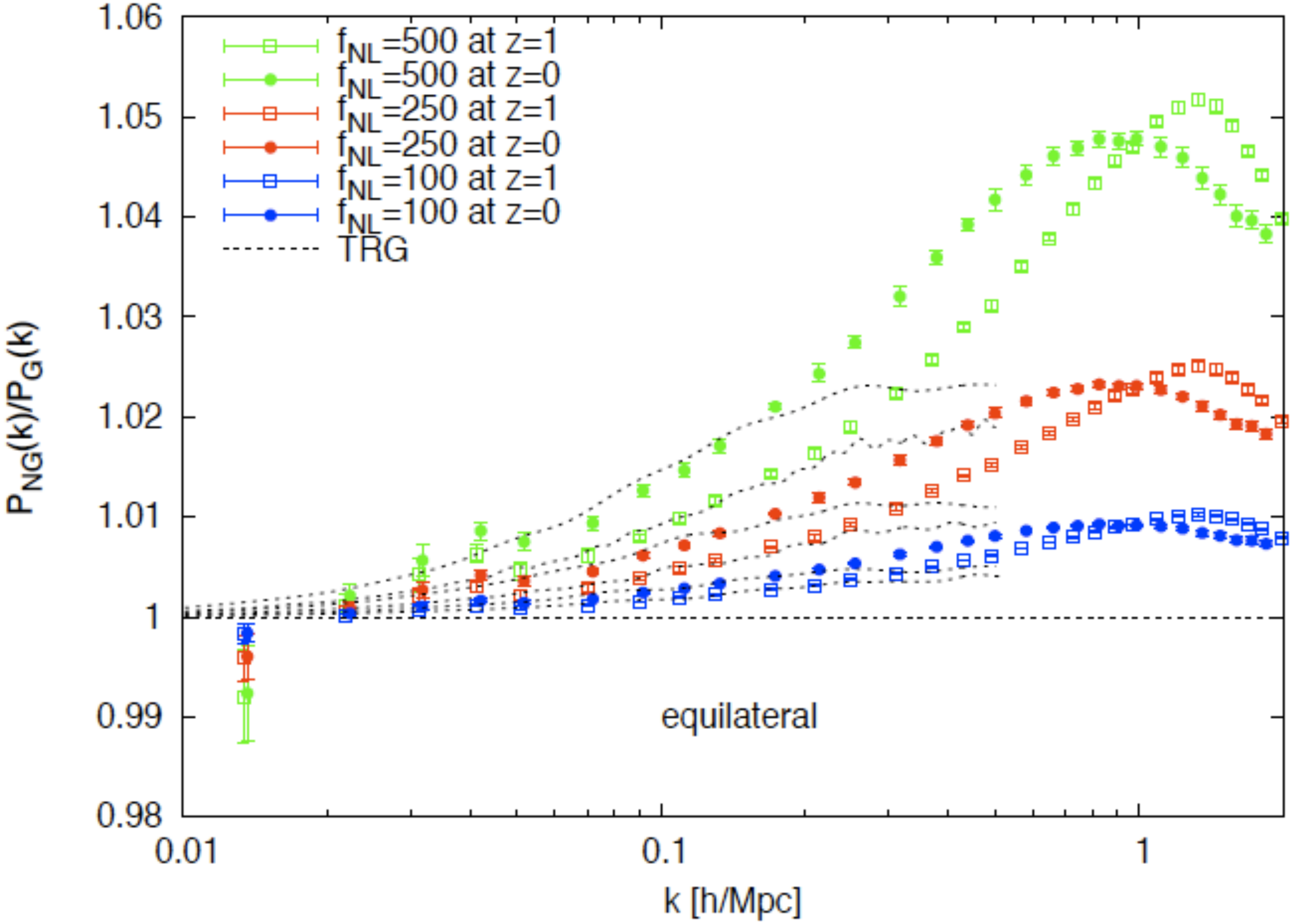
Bartolo, Beltrán Almeida, Matarrese, Pietroni & Riotto 2010

Comparison of RG with N-body simulations (local case)



from: Wagner, Verde & Boubekeur 2010

Comparison of RG with N-body simulations (equilateral case)



from: Wagner, Verde & Boubekeur 2010

Searching for non-Gaussianity with rare events

- Besides using standard statistical estimators, like (mass) bispectrum, trispectrum, three and four-point function, skewness, etc. …, one can look at the tails of the distribution, i.e. at rare events.

- Rare events have the advantage that they often maximize deviations from what predicted by a Gaussian distribution, but have the obvious disadvantage of being rare! But remember that, according to Press-Schechter-like schemes, all collapsed DM halos correspond to (rare) peaks of the underlying density field.

- Matarrese, Verde & Jimenez (2000) and Verde, Jimenez, Kamionkowski & Matarrese showed that clusters at high redshift (z>1) can probe NG down to $f_{NL} \sim 10^2$

- Alternative approach by LoVerde et al. (2007). Determination of mass function using stochastic approach (first-crossing of a diffusive barrier) Maggiore & Riotto 2009. Ellispsoidal collapse used by Lam & Sheth 2009. Saddle-point + diffusive barrier (Paranjape et al. 2010). Log-Edgeworth expantion: LoVerde & Smith 2011. Excursion sets studied with correlated steps: Paranjape, Lam & Sheth 2011; Paranjape & Sheth 2011.

- Excellent agreement of analytical formulae with N-body simulations found by Grossi et al. 2009 ... and many others.

Different approaches to the NG halo mass function

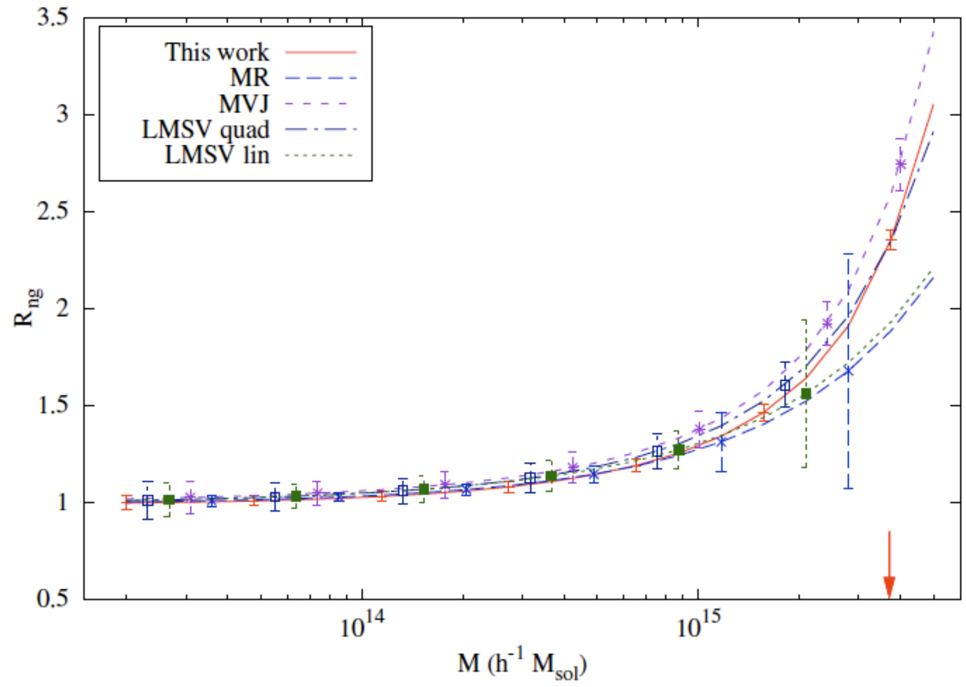


Figure 6: Same as Fig. 5, but including filter effects. These affect only the error bars for MVJ and LMSV, and they affect both the curve and the error bars for MR and our result. For MR and our result, the Gaussian mass function used to construct the ratio $R_{\rm ng}$, is taken as the non-Gaussian result at $f_{\rm NL} = 0$, and hence includes filter effects.

from: D'Amico et al. 2010

NG vs. Halo Mass Function

- Relevant effects:
  - non-Markovianity, already there in Gaussian case, unavoidable in NG case
  - non-spherical collapse
  - connecting random walks w. DM halos
  - diffusive collapse threshold?
- Dealing with rare events i.e. tails of NG distribution
- Validation with N-body simulations crucial (although very rare events/tails not probed by finite number of realizations → analytical treatments welcome!)
- Understanding/definition of connection between analytical/numerical quantities and real observables → to what level is this affecting NG (e.g. f_{NL}) measurements?

DM halo clustering as (the most stringent?) constraint on NG

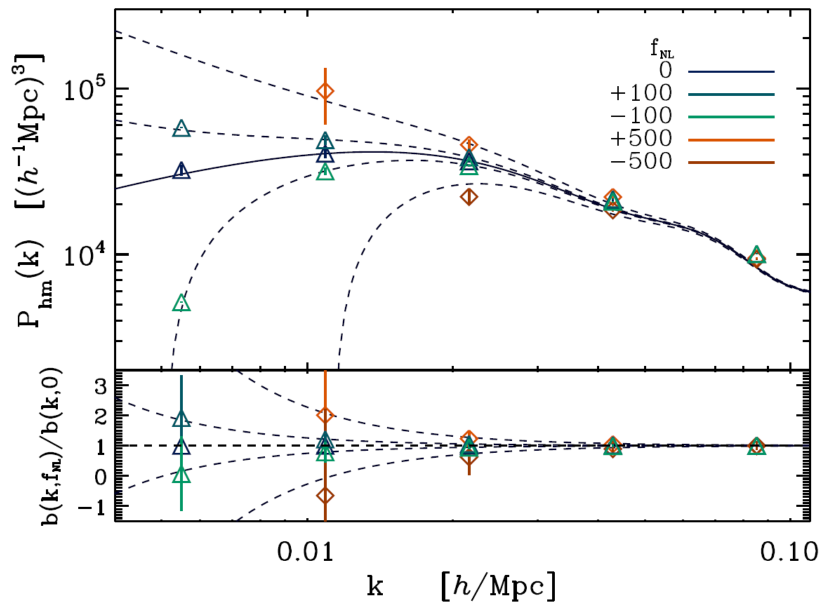
$$\delta_{halo} = b\, \delta_{matter}$$

Dalal et al. (2007) have shown that halo bias is sensitive to primordial non-Gaussianity through a scale-dependent correction term

$$\Delta b(k)/b \;\alpha\; 2\, f_{NL} \delta_c / k^2$$

This opens interesting prospects for constraining or measuring NG in LSS but demands for an accurate evaluation of the effects of (general) NG on halo biasing.

Dalal, Dore', Huterer & Shirokov 2007



Clustering of peaks (DM halos) of NG density field

Start from results obtained in the 80's by

Grinstein & Wise 1986, ApJ, 310, 19

Matarrese, Lucchin & Bonometto 1986, ApJ, 310, L21

giving the general expression for the peak 2-point function as a function of N-point connected correlation functions of the background linear (i.e. Lagrangian) mass-density field

$$\xi_{h,M}(|\mathbf{x}_1 - \mathbf{x}_2|) = -1 +$$

$$\exp\left\{\sum_{N=2}^{\infty}\sum_{j=1}^{N-1}\frac{\nu^N\sigma_R^{-N}}{j!(N-1)!}\xi^{(N)}\left[\underset{j\,times}{\mathbf{x}_1,\ldots,\mathbf{x}_1},\ \underset{(N-j)\,times}{\mathbf{x}_2,\ldots\ldots,\mathbf{x}_2}\right]\right\}$$

(requires use of path-integral, cluster expansion, multinomial theorem and asymptotic expansion). The analysis of NG models was motivated by a paper by Vittorio, Juszkiewicz and Davis (1986) on bulk flows.

THE ASTROPHYSICAL JOURNAL, 310:L21–L26, 1986 November 1

A PATH-INTEGRAL APPROACH TO LARGE-SCALE MATTER DISTRIBUTION ORIGINATED BY NON-GAUSSIAN FLUCTUATIONS

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ABSTRACT

The possibility that, in the framework of a biased theory of galaxy clustering, the underlying matter distribution be non-Gaussian itself, because of the very mechanisms generating its present status, is explored. We show that a number of contradictory results, seemingly present in large-scale data, in principle can recover full coherence, once the requirement that the underlying matter distribution be Gaussian is dropped. For example, in the present framework the requirement that the two-point correlation functions vanish at the same scale (for different kinds of objects) is overcome. A general formula, showing the effects of a non-Gaussian background on the expression of three-point correlations in terms of two-point correlations, is given.
Subject heading: galaxies: clustering

THE ASTROPHYSICAL JOURNAL, 310:19–22, 1986 November 1

NON-GAUSSIAN FLUCTUATIONS AND THE CORRELATIONS OF GALAXIES OR RICH CLUSTERS OF GALAXIES[1]

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ABSTRACT

Natural primordial mass density fluctuations are those for which the probability distribution, for mass density fluctuations averaged over the horizon volume, is independent of time. This criterion determines that the two-point correlation of mass density fluctuations has a Zeldovich power spectrum (i.e., a power spectrum proportional to k at small wavenumbers) but allows for many types of reduced (connected) higher correlations. Assuming galaxies or rich clusters of galaxies arise wherever suitably averaged natural mass density fluctuations are unusually large, we show that the two-point correlation of galaxies or rich clusters of galaxies can have significantly more power at small wavenumbers (e.g., a power spectrum proportional to $1/k$ at small wavenumbers) than the Zeldovich spectrum. This behavior is caused by the non-Gaussian part of the probability distribution for the primordial mass density fluctuations.
Subject headings: cosmology — galaxies: clustering

Halo bias in NG models

- Matarrese & Verde 2008 applied this relation to the case of NG of the gravitational potential, obtaining the power-spectrum of dark matter halos modeled as high "peaks" (up-crossing regions) of height $\nu=\delta_c/\sigma_R$ of the underlying mass density field (Kaiser's model). Here $\delta_c(z)$ is the critical overdensity for collapse (at redshift a) and σ_R is the *rms* mass fluctuation on scale R ($M \sim R^3$).

- Account for motion of peaks (going from Lagrangian to Eulerian space), which implies (Catelan et al. 1998)

$$1+\delta_h(\mathbf{x}_{Eulerian}) = (1+\delta_h(\mathbf{x}_{Lagrangian}))(1+\delta_R(\mathbf{x}_{Eulerian}))$$

and (to linear order) $b=1+b_L$ (Mo & White 1996) to get the scale-dependent halo bias in the presence of NG initial conditions. *Corrections may arise from second-order bias and GR terms.*

- Alternative approaches (e.g. based on 1-loop calculations) by Taruya et al. 2008; Matsubara 2009; Jeong & Komatsu 2009. Giannantonio & Porciani 2010 improve fit to N-body simulations by assuming dependence on gravitational potential) → extension to bispectrum by Baldauf et al. 2011

Halo bias in NG models

- Extension to general (scale and configuration dependent) NG is straightforward
- In full generality write the ϕ bispectrum as $B_\phi(k_1,k_2,k_3)$. The relative NG correction to the halo bias is

$$\frac{\Delta b_h}{b_h} = \frac{\Delta_c(z)}{D(z)} \frac{1}{8\pi^2\sigma_R^2} \int dk_1 k_1^2 \mathcal{M}_R(k_1) \times \int_{-1}^{1} d\mu \mathcal{M}_R\left(\sqrt{\alpha}\right) \frac{B_\phi(k_1, \sqrt{\alpha}, k)}{P_\phi(k)} \times \frac{1}{M_R(k)}$$

$$\alpha = k_1^2 + k^2 + 2k_1 k\mu$$

- It also applies to non-local (e.g. "equilateral") NG (DBI, ghost inflation, etc..) and universal NG term!! (→ see also Schmidt & Kamionkowski 2010).
- Calibrated to N-body simulations by Grossi et al. (2009), Desjacques et al. 2009; Pillepich et al. 2009; ...

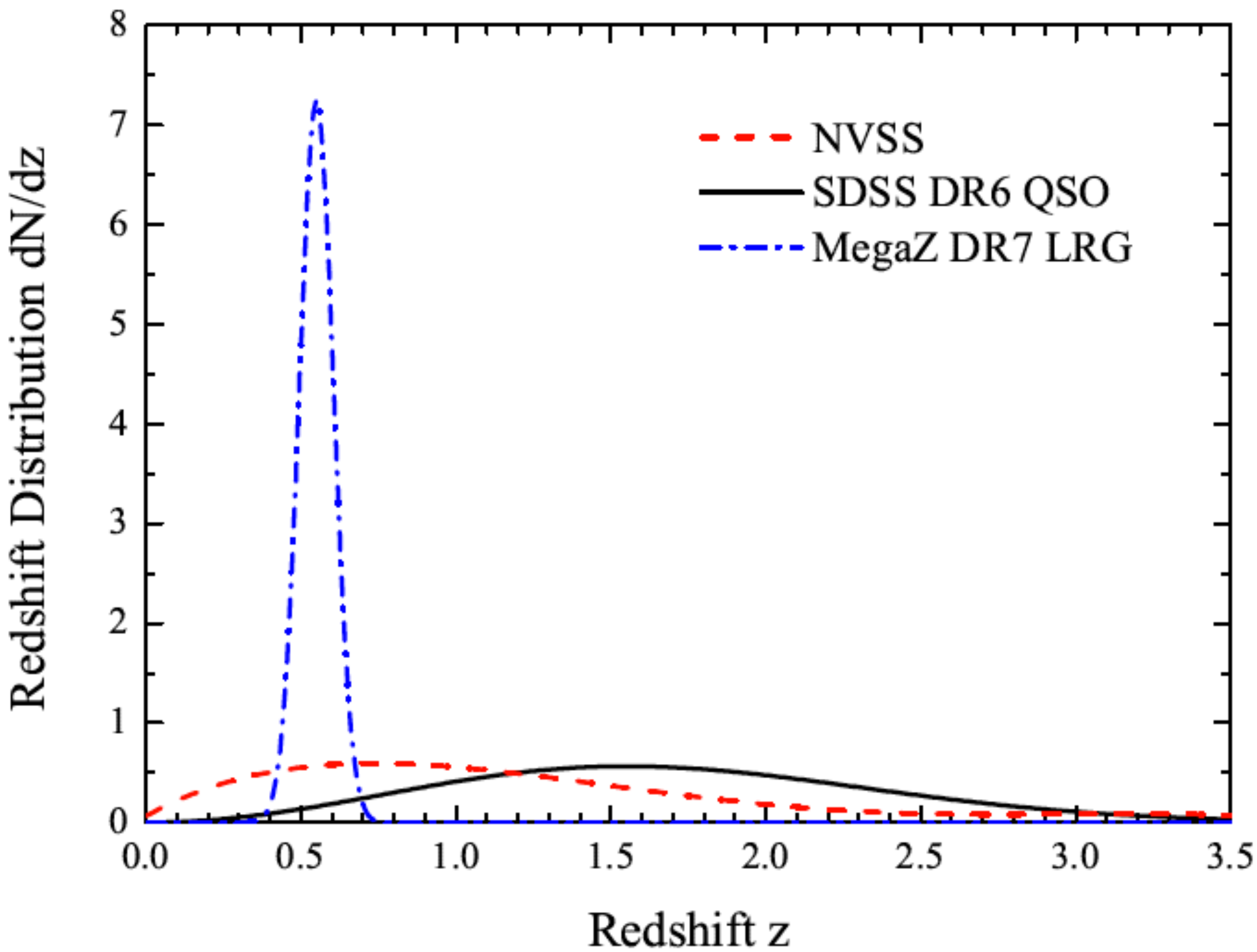
Observational status

Data/method	$f_{\rm NL}$ (local-type 95% CL)	reference **ad ref to bibl**
Photometric LRG - bias	$63^{+54+101}_{-85-331}$	Slosar et al. 2008
Spectroscopic LRG- bias	$70^{+74+139}_{-83-191}$	Slosar et al. 2008
QSO - bias	8^{+26+47}_{-37-77}	Slosar et al. 2008
combined	28^{+23+42}_{-24-57}	Slosar et al. 2008
NVSS–ISW	$105^{+647+755}_{-337-1157}$	Slosar et al. 2008
NVSS–ISW	$236 \pm 127 (2-\sigma)$	Afshordi&Tolley 2008
NVSS-ACF (bias + ISW)	$10 < f_{\rm NL} < 106$	Xia et al. 2010
SDSS DR6 QSOs (bias + ISW)	58 ± 24 (1 sigma)	Xia et al. 2010

→ Xia et al. (2011) analysis in terms of C_l: *$5 < f_{\rm NL} < 84$ (2 sigma)*

Constraints on NG from high-z probes

Xia, Baccigalupi, Matarrese, Verde & Viel arXiv:1104.5015v2 [astro-ph.CO]



Angular power-spectra

Xia, Baccigalupi, Matarrese, Verde & Viel **arXiv:1104.5015v2 [astro-ph.CO]**

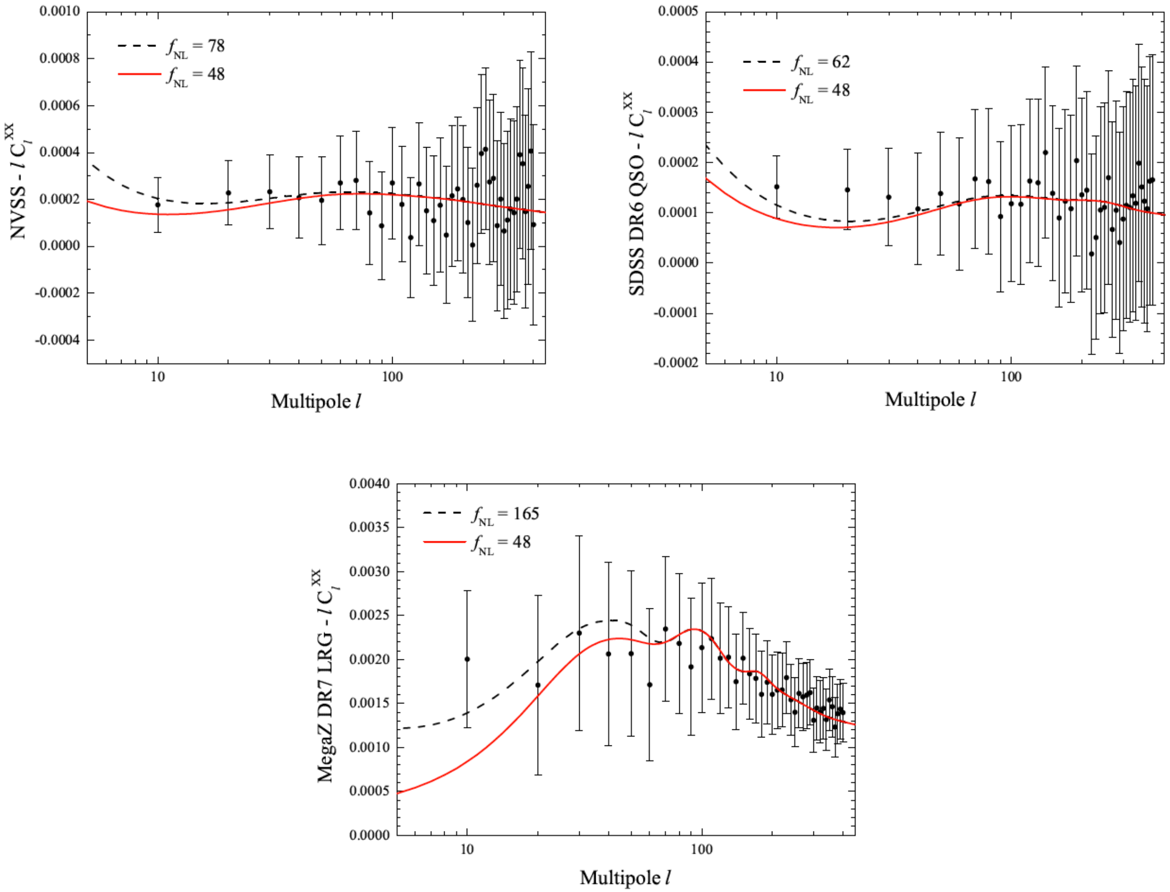


FIG. 2: Observed angular power spectra of three LSS tracers. The black dashed lines are the best fit models using angular power spectra only. The red solid lines are the best fit models $f_{\rm NL} = 48$ when using all data together. For illustration purposes, we show the binned power spectra with the bin size $\Delta\ell = 10$.

Angular Cross-Spectra

Xia, Baccigalupi, Matarrese, Verde & Viel arXiv:1104.5015v2 [astro-ph.CO]

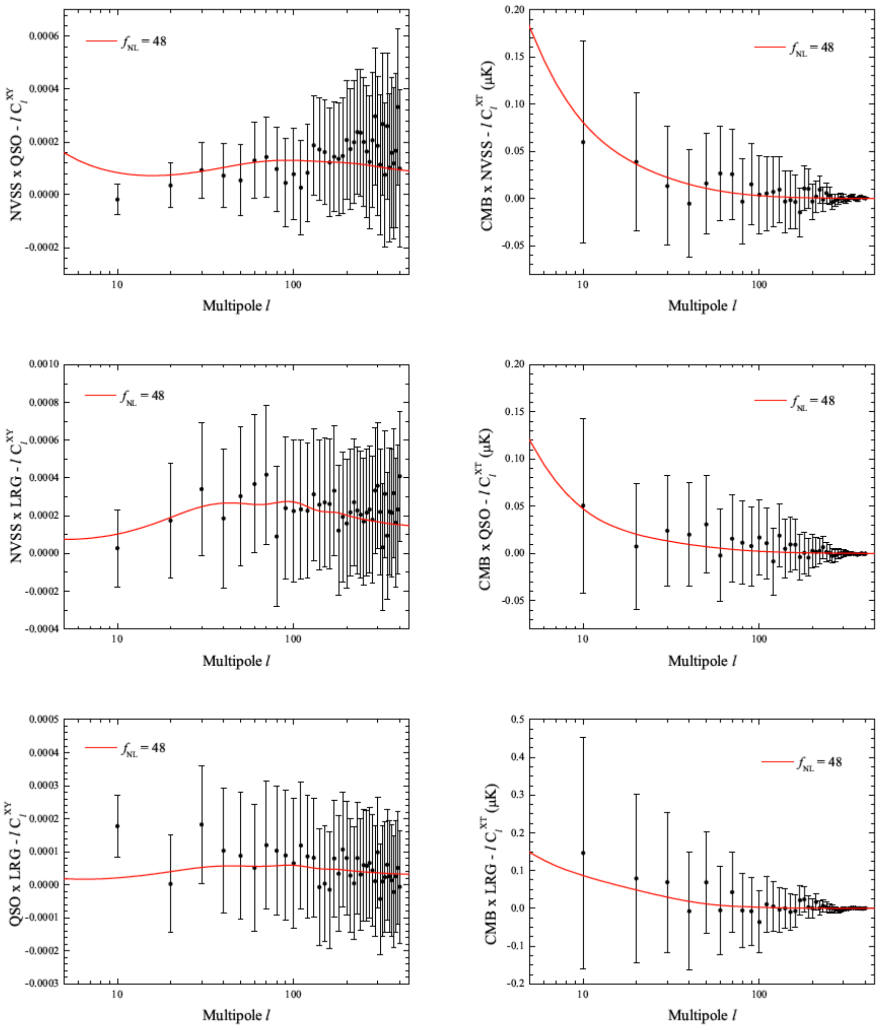


FIG. 3: Left Panels: Observed cross-correlation power spectra among three LSS tracers. Right Panels: Observed cross-correlation power spectra between CMB ILC map and three LSS tracers. The red solid lines are the best fit models $f_{NL} = 48$ when using all data together. For illustration purposes, we show the binned power spectra with the bin size $\Delta\ell = 10$.

Constraints on Primordial NG

Xia, Baccigalupi, Matarrese, Verde & Viel [arXiv:1104.5015v2 [astro-ph.CO]]

Datasets	Non-Gaussianity	
WMAP7+BAO+SN	$1\,\sigma$	$2\,\sigma$
Local Type $f_{\rm NL}$		
$+C_\ell^{\rm XX}$(NVSS)	78 ± 52	$[-34, 187]$
$+C_\ell^{\rm XX} + C_\ell^{\rm XT}$(NVSS)	74 ± 40	$[-16, 166]$
$+C_\ell^{\rm XX}$(QSO)	62 ± 26	$[5, 115]$
$+C_\ell^{\rm XX} + C_\ell^{\rm XT}$(QSO)	59 ± 21	$[17, 103]$
$+C_\ell^{\rm XX}$(LRG)	165 ± 105	$[-81, 351]$
$+C_\ell^{\rm XX} + C_\ell^{\rm XT}$(LRG)	153 ± 95	$[-51, 347]$
$+C_\ell^{\rm XX}$(ALL)	68 ± 22	$[22, 108]$
$+C_\ell^{\rm XX} + C_\ell^{\rm XY} + C_\ell^{\rm XT}$(ALL)	48 ± 20	$[5, 84]$
Equilateral Template $\tilde{f}_{\rm NL}$		
$+C_\ell^{\rm XX} + C_\ell^{\rm XY} + C_\ell^{\rm XT}$(ALL)	50 ± 265	$[-419, 625]$
Enfolded Template $\tilde{f}_{\rm NL}$		
$+C_\ell^{\rm XX} + C_\ell^{\rm XY} + C_\ell^{\rm XT}$(ALL)	183 ± 95	$[-12, 358]$
Cubic Correction $g_{\rm NL} \times 10^{-5}$		
$+C_\ell^{\rm XX} + C_\ell^{\rm XY} + C_\ell^{\rm XT}$(ALL)	5.7 ± 3.0	$[-1.2, 11.3]$

Note: Multipoles with l<10 have been excluded in the analysis. Reintroducing them and allowing for a constraint on number yields back Xia et al. 2010 results for NVSS

Note: For enfolded and equilateral we used the template, but
... see Licia's talk

for the orthogonal template: $\tilde{f}_{\rm NL}^{\rm orth} = -92 \pm 47 \ (1\,\sigma)$ and $-179 < \tilde{f}_{\rm NL}^{\rm orth} < 6 \ (2\,\sigma)$

Observational prospects

Data/method	$\Delta f_{\rm NL}$ $(1-\sigma)$	reference
BOSS–bias	18	Carbone et al 2008
ADEPT/Euclid–bias	1.5	Carbone et al 2008
PANNStarrs –bias	3.5	Carbone et al 2008
LSST–bias	0.7	Carbone et al 2008
LSST-ISW	7	Afshordi& Tolley 2008
BOSS–bispectrum	35	Sefusatti & Komatsu 2008
ADEPT/Euclid –bispectrum	3.6	Sefusatti & Komatsu 2008
Planck-Bispectrum	3	Yadav et al . 2007
BPOL-Bispectrum	2	Yadav et al . 2007

Can we test standard single-field inflation NG?

- GR contributions to f_{NL} are universally present and can be seen through their effect on halo biasing (Verde & Matarrese 2009)

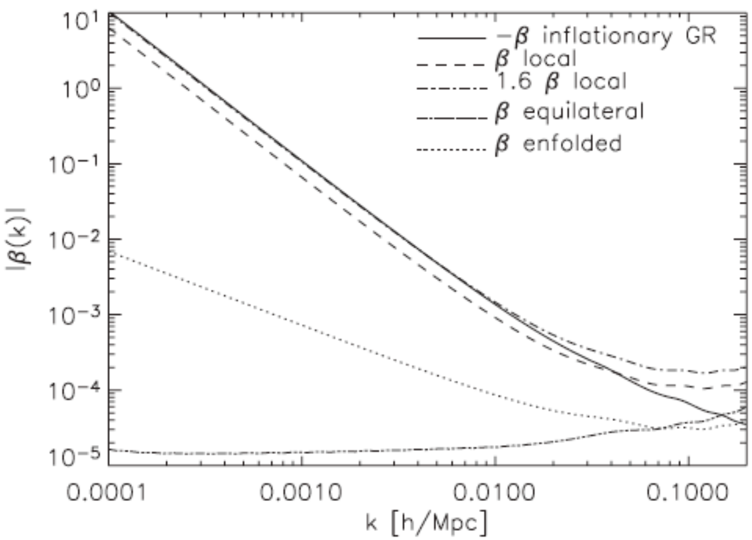


Figure 1. Scale dependence of the large-scale halo bias induced by a non-zero bispectrum, indicated by the β function of Equation (3) for the four types of non-Gaussianity discussed in the text. The solid line shows the absolute value of β for the inflationary, GR correction large-scale structure bispectrum. Note that the quantity is actually negative. The dashed line shows β for the local type of primordial non-Gaussianity for $f_{NL}^{loc} = 1$ (the quantity is positive). It is clear that the scale-dependent bias effect due to the inflationary bispectrum mimics a local primordial non-Gaussianity with effective $f_{NL} \sim -1$ at $k > 0.02H/\mathrm{Mpc}$ and ~ -1.6 for $k < 0.01h/\mathrm{Mpc}$. The dot-dot-dot-dashed line shows the effect of equilateral non-Gaussianity for $f_{NL}^{eq} = 1$ and the dotted line shows the enfolded type with $f_{NL}^{enf} = 1$.

$$f_{NL}^{infl,GR}(k_i, k_j, k_k) = -\frac{5}{3}\left[1 - \frac{5}{2}\frac{k_i k_j \cos\theta_{ij}}{k_k^2}\right]$$

Table 1
Forecasted Non-Gaussianity Constraints

Type NG	CMB Bispectrum		Halo Bias	
	Planck	(CM)BPol	Euclid	LSST
	1σ errors			
Local	$3^{(A)}$	$2^{(A)}$	$1.5^{(B)}$	$0.7^{(B)}$
Equilateral	$25^{(C)}$	$14^{(C)}$	...	...
Enfolded	$\mathcal{O}10$	$\mathcal{O}10$	$39^{(E)}$	$18^{(E)}$
	$\#\sigma$ Detection			
GR	N/A	N/A	$1^{(E)}$	$2^{(E)}$
Secondaries	$3^{(F)}$	$5^{(F)}$	N/A	N/A

References. (1) Yadav et al. 2007, (2) Carbone et al. 2008, (3) Baumann et al. 2009; Sefusatti et al. 2009, (4) this work, (5) e.g., Mangilli & Verde 2009.

Conclusions

- ☞ *Contrary to earlier naive expectations, some level of non-Gaussianity is generically present in all inflation models. The level of non-Gaussianity predicted in the simplest (single-field, slow-roll, canonical kinetic term, BD initial state) inflation is slightly below the minimum value detectable by Planck and at reach of future galaxy surveys (accounting for GR effects)*

- ☞ Constraining/detecting non-Gaussianity is a powerful tool to discriminate among competing scenarios for perturbation generation (*standard slow-roll inflation, curvaton, modulated-reheating, DBI, ghost inflation, multi-field, etc. …*) some of which imply large non-Gaussianity. Non-Gaussianity will soon become the *smoking-gun* for non-standard inflation models.